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Top Quark Mass Prediction in Superstring Derived Standard–like Models

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Abstract

A remarkable achievement of the realistic superstring standard–like models is the successful prediction of the top quark mass, assuming the Minimal Supersymmetric Standard Model spectrum below the string scale. Recently it was shown that string scale unification requires the existence of additional matter, in vector–like representations, at intermediate energy scales and that certain string models contain the needed representations in their massless spectrum. I obtain the top, bottom and tau lepton Yukawa couplings in these models, by calculating tree level string amplitudes, in terms of the unified gauge coupling and certain vacuum expectation values that are required for the consistency of the string models. Using two–loop renormalization group equations for the gauge and Yukawa couplings, I study the effect of the intermediate matter thresholds on the top quark mass prediction. Agreement with the experimental values of $\alpha_{\rm strong}(M_Z)$, $\sin^2\theta_W(M_Z)$ and $\alpha^{-1}_{\rm em}(M_Z)$ is imposed. It is found that the physical top quark mass prediction is increased to the range $192-200$ GeV and that the ratio $\lambda_b(M_Z)/\lambda_\tau(M_Z)$ is in agreement with experiment.

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One of the intriguing successes of the realistic superstring standard–like models in the free fermionic formulation is the successful prediction of the top quark mass. In Ref. [1] the top quark mass was predicted to be in the mass range

$$m_t \approx 175 - 180 \text{ GeV} , \tag{1}$$

three years prior to its experimental observation. Remarkably, this prediction is in agreement with the top quark mass as observed by the recent CDF and D0 collaborations [3]. In obtaining the top quark mass prediction, it was assumed in Ref. [1] that the spectrum below the string scale is that of the Minimal Supersymmetric Standard Model (MSSM), i.e. three generations plus two Higgs doublets. However, it was recently shown [4] that string gauge coupling unification requires the existence of intermediate matter thresholds, beyond the MSSM spectrum. This additional matter takes the form of additional color triplets and electroweak doublets, in vector–like representations, with specific weak hypercharge assignments. Remarkably, the same string standard–like models that led to the prediction, Eq. (1), allow for the existence of the needed additional states and the required weak hypercharge assignments, to achieve string scale unification.

In this paper, I investigate the effect of the intermediate matter thresholds on the top quark mass prediction in the realistic superstring derived standard–like models. In these models the top, bottom and tau lepton Yukawa couplings are calculated by evaluating the string tree level amplitudes between the vertex operators in the effective conformal field theory. The Yukawa couplings are obtained in terms of the unified gauge coupling and certain VEVs that are required for the consistency of the string models. The gauge and Yukawa couplings are extrapolated numerically from the string unification scale to low energies by using the coupled two–loop supersymmetric Renormalization Group Equations (RGEs), including the contribution of the extra matter thresholds. Agreement with $\alpha_{\text{strong}}(M_Z)$, $\sin^2\theta_W(M_Z)$ and $\alpha_{\text{em}}(M_Z)$ is imposed. It is found that the running top quark mass is shifted to $m_t \approx 185 - 190$ GeV and the physical top quark mass is in the range $192 - 200$ GeV. The string models under consideration also predict $\lambda_b = \lambda_\tau$ at the string unification scale. Using the same extrapolation, it is found that for a large portion of the parameter space this

prediction is in agreement with the experimental value of $m_b(M_Z)/m_\tau(M_Z)$. Thus, I show that LEP precision data for $\alpha_{\rm strong}$ and $\sin^2\theta_W$ as well as the CDF/D0 top quark observation and the b/τ mass relation can all simultaneously be consistent with the superstring derived standard–like models. Here I present the main results. A detailed account of the numerical results and details of the string calculations of the Yukawa couplings will be given elsewhere [2].

One of the fundamental mysteries of the observed fermion mass spectrum is the large mass splitting between the top quark and the lighter quarks and leptons. Especially difficult to understand within the context of the Standard Model, and its field theoretic extensions, is the big splitting in the heaviest generation. The superstring derived standard–like models suggest a superstring mechanism that explains the suppression of the lighter quarks and leptons masses relative to the top quark mass. At the cubic level of the superpotential only $+\frac{2}{3}$ charged quarks get nonvanishing Yukawa couplings, while the remaining quarks and leptons get their mass terms from nonrenormalizable terms. This selection mechanism, between $+\frac{2}{3}$ and $-\frac{1}{3}$ Yukawa couplings, results from the specific assignments of boundary conditions that specify the string models. Due to the horizontal symmetries of the string models, each of the chiral generations couples at the cubic level to different doublet Higgs multiplets. Only one pair of the Higgs doublets remains light at low energies [5]. As a result only one nonvanishing mass term, namely the top quark mass term, remains at low energies. The mass terms for the lighter quarks and leptons are obtained from nonrenormalizable terms. The nonrenormalizable terms have the general form,

$$cgf_if_jh(\phi/M)^{n-3} \tag{2}$$

where c are the calculable coefficients of the $n^{\rm th}$ order correlators, g is the gauge coupling at the unification scale, f_i, f_j are the quark and lepton fields, h are the light Higgs representations, and ϕ are Standard Model singlets in the massless spectrum of the string models. An important property of the superstring standard–like models is the absence of gauge and gravitational anomalies apart from a single "anomalous $U(1)$" symmetry. This anomalous $U(1)_A$ generates a Fayet–Iliopoulos term that breaks supersymmetry at the Planck scale [6]. Supersymmetry is restored and $U(1)_A$

is broken by giving VEVs to a set of Standard Model singlets in the massless string spectrum along the flat F and D directions [6]. However, as the charge of these singlets must have $Q_A < 0$ to cancel the anomalous $U(1)$ D–term equation, in many models a phenomenologically realistic solution does not exist. The only models that were found to admit a solution are models which have cubic level Yukawa couplings only for $+\frac{2}{3}$ charged quarks. The magnitude of these VEVs is set by the Fayet–Iliopoulos D–term, which is generated due to the "anomalous" $U(1)_A$ at the one–loop level in string perturbation theory. Consequently, some of the nonrenormalizable, order N terms, become effective renormalizable terms with effective Yukawa couplings, $\lambda = cg(\langle\phi\rangle/M)^{n-3}$.

The superstring standard–like models are constructed in the free fermionic formulation [7]. A model is generated by a consistent set of boundary condition basis vectors. The physical spectrum is obtained by applying the generalized GSO projections. Each physical state is described in terms of vertex operators in the effective conformal field theory. Quark and lepton mass terms are obtained by calculating the correlators

$$A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle, \tag{3}$$

between the vertex operators. The nonvanishing correlators must be invariant under all the symmetries of a given string model and satisfy all the string selection rules. Consequently, most of the quark and lepton mass terms vanish at the cubic level of the superpotential. One must then examine whether potential quark and lepton mass terms can be obtained from nonrenormalizable terms.

The first five basis vectors consist of the NAHE set, $\{\mathbf{1}, S, b_1, b_2, b_3\}$ [8]. At the level of the NAHE set the gauge group is $SO(10) \times SO(6)^3 \times E_8$, with 48 generations. The number of generations is reduced to three and the $SO(10)$ gauge group is broken to $SU(3) \times SU(2) \times U(1)^2$ by adding to the NAHE set three additional basis vectors, $\{\alpha, \beta, \gamma\}$. The basis vector that breaks the $SO(2n)$ symmetry to $SU(n) \times U(1)$ must contain half integral boundary conditions for the world–sheet complex fermions that generate the $SO(10)$ symmetry. This basis vector plays an important role in the superstring selection mechanism and will be denoted as the vector γ.

The two basis vectors $\{\mathbf{1}, S\}$ produce a model with $N = 4$ space–time supersymmetry and $SO(44)$ gauge group. At this level all of the world–sheet fermions are equivalent. The NAHE set plus the vector 2γ divide the world–sheet fermions into several groups. The six left–moving real fermions, $\chi^{1,\cdots,6}$ are paired to form three complex fermions denoted χ^{12}, χ^{34} and χ^{56}. The sixteen right–moving complex fermions $\bar{\psi}^{1\cdots5}\bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1,\cdots,8}$ produce the observable and hidden gauge groups, that arise from the sixteen dimensional compactified space of the heterotic string in ten dimensions. Finally, the twelve left–moving $\{y, \omega\}^{1\cdots6}$ and twelve right–moving $\{\bar{y}, \bar{\omega}\}^{1\cdots6}$ real fermions correspond to the left/right symmetric internal conformal field theory of the heterotic string. The assignment of boundary conditions in the vector γ to this set of internal world–sheet fermions selects cubic level Yukawa couplings for $+\frac{2}{3}$ or $-\frac{1}{3}$ charged quarks.

Each of the sectors b_1, b_2 and b_3 produce one generation. Three pairs of electroweak Higgs doublets $\{h_1, h_2, h_3, \bar{h}_1, \bar{h}_2, \bar{h}_3\}$ are obtained from the Neveu–Schwarz sector. One or two additional pairs, $\{h_{45}, \bar{h}_{45}\}$ are obtained from a combination of the additional basis vectors. The three boundary condition basis vectors $\{\alpha, \beta, \gamma\}$ break the horizontal $SO(6)^3$ symmetries to factors of $U(1)$s. Three $U(1)$ symmetries arise from the complex right–moving fermions $\bar{\eta}^1$, $\bar{\eta}^2$, $\bar{\eta}^3$. Additional horizontal $U(1)$ symmetries arise by pairing two of the right–moving real internal fermions $\{\bar{y}, \bar{\omega}\}$. For every right–moving $U(1)$ symmetry, there is a corresponding left–moving global $U(1)$ symmetry that is obtained by pairing two of the left–moving real fermions $\{y, \omega\}$. Each of the remaining world–sheet internal fermions from the set $\{y, \omega\}$ is paired with a right–moving real internal fermions from the set $\{\bar{y}, \bar{\omega}\}$ to form a Ising model operator.

The assignment of boundary conditions in the basis vector γ for the internal world–sheet fermions, $\{y, \omega | \bar{y}, \bar{\omega}\}$ selects a cubic level mass term for $+\frac{2}{3}$ or $-\frac{1}{3}$ charged quarks. For each of the sectors b_1, b_2 and b_3 the fermionic boundary conditions select the cubic level Yukawa couplings according to the difference,

$$\Delta_j = |\gamma(L_j) - \gamma(R_j)| = 0, 1 \tag{4}$$

where $\gamma(L_j)/\gamma(R_j)$ are the boundary conditions in the vector γ for the internal world–

sheet fermions from the set $\{y, \omega | \bar{y}, \bar{\omega}\}$, that are periodic in the vector b_j. If $\Delta_j = 1$ then a Yukawa coupling for the $+\frac{2}{3}$ charged quark from the sector b_j is nonzero and the Yukawa coupling for the $-\frac{1}{3}$ charged quark vanishes. The opposite occurs if $\Delta_j = 0$. Thus, the states from each of the sectors b_1, b_2 and b_3 can have a cubic level Yukawa coupling for the $+\frac{2}{3}$ or $-\frac{1}{3}$ charged quark, but not for both. We can construct string models in which both $+\frac{2}{3}$ and $-\frac{1}{3}$ charged quarks get a cubic level mass term. The model of ref. [9] is an example of such a model. By contrast, we can also construct string models in which only $+\frac{2}{3}$ charged quarks get a nonvanishing cubic level mass term. The model of ref. [1] is an example of such a model. In Ref. [10] this selection rule is proven by using the string consistency constraints and Eq. (4) to show that either the $+\frac{2}{3}$ or the $-\frac{1}{3}$ mass term is invariant under the $U(1)_j$ symmetry.

Due to the horizontal $U(1)$ symmetries the states from each of the sectors b_j, $(j = 1, 2, 3)$ can couple at the cubic level only to one of the Higgs pairs h_j, $\bar{h}_j$. This results due to the fact that the states from a sector b_j and the Higgs doublets h_j and $\bar{h}_j$ are charged with respect to one of the horizontal $U(1)_j$, $j = 1, 2, 3$ symmetries. Therefore, the basis vectors of Ref. [1] lead to the following nonvanishing cubic level mass terms for the states from the sectors b_1, b_2 and b_3,

$$\{u^c_{L_i} Q_i \bar{h}_i + N^c_{L_i} L_i \bar{h}_i\} \quad (i = 1, 2, 3), \tag{5}$$

where the common normalization factor is obtained by evaluating the correlators, Eq. (3), of the $N = 3$ order terms in Eq. (5), yielding $A_3 = \sqrt{2}g$. A very restricted class of standard–like models with $\Delta_j = 1$ for $j = 1, 2, 3$, were found to admit a solution to the F and D flatness constraints. Consequently, in these models only $+\frac{2}{3}$ charged quarks obtain a cubic level mass term. Mass terms for $-\frac{1}{3}$ charged quarks and for charged leptons must arise from nonrenormalizable terms. In the model of Ref. [1] the following nonvanishing mass terms are obtained at the quartic order,

$$\begin{aligned} W_4 = \{ \quad & d^c_{L_1} Q_1 h_{45} \Phi_1 + e^c_{L_1} L_1 h_{45} \Phi_1 + \\ & d^c_{L_2} Q_2 h_{45} \bar{\Phi}_2 + e^c_{L_2} L_2 h_{45} \bar{\Phi}_2 \}. \end{aligned} \tag{6}$$

The quartic term coefficients are obtained by calculating the $N = 4$ order correlators and are equal to $gI/\sqrt{\pi}M_{\rm Pl}$. I is a one dimensional complex integral which is evaluated numerically, $I \approx 77.7$.

From Eq. (6) we observe that if some of the Standard Model singlets, that appear in the quartic order terms, acquire a VEV by the cancelation of the anomalous $U(1)$ D–term equation, then effective mass terms for the $-\frac{1}{3}$ quarks and for charged leptons are obtained. At the same time an analysis of the renormalizable and nonrenormalizable superpotential suggests that only one pair of Electroweak Higgs doublets remains light at low energies [5]. For typical scenarios, those consist of $\bar{h}_1$ or $\bar{h}_2$ and h_{45}. Because the states from each of the sectors b_1, b_2 and b_3 can couple only to one of the Higgs pairs, at the cubic level of the superpotential only one mass term remains. Therefore, at the cubic level of the superpotential only the top quark has a nonvanishing mass term. The top quark Yukawa coupling is therefore given by

$$\lambda_t(M_{\rm string}) = g\sqrt{2} \tag{7}$$

where g is the gauge coupling at the unification scale.

In ref. [1] a solution to the F and D flatness constraints was found with,

$$|\langle\bar{\Phi}_2\rangle|^2 = \frac{g^2}{16\pi^2}\frac{1}{2\alpha'} \tag{8}$$

where α' is the string tension, $\alpha' = 16\pi/g^2 M_{pl}^2$ [11]. Thus, after inserting the VEV of $\bar{\Phi}_2$, the effective bottom quark and tau lepton Yukawa couplings are given by,

$$\lambda_b = \lambda_\tau = 0.35g^3. \tag{9}$$

The top quark mass prediction is obtained by taking $g \sim 1/\sqrt{2}$ at the unification scale. The three Yukawa couplings are run to the low energy scale by using the MSSM one–loop RGEs. The bottom quark mass, $m_b(M_Z)$ and the W-boson mass, $M_W(M_W)$ are used to fix the two VEVs, v_1 and v_2. Using the relation,

$$m_t \approx \lambda_t(M_Z)\sqrt{\frac{2M_W^2}{g_2^2(M_W)} - \left(\frac{m_b(M_Z)}{\lambda_b(m_Z)}\right)^2} \tag{10}$$

the top quark mass prediction, Eq. (1), is obtained.

Eq. (1) was obtained in ref. [1], assuming the MSSM spectrum below the string unification scale, $M_{\rm string} \approx g_{\rm string} \times 5 \times 10^{17}$ GeV, where $g_{\rm string}$ is the gauge coupling at the string unification scale. However, this assumption results in disagreement with the values extracted at LEP for $\alpha_{\rm strong}(M_Z)$ and $\sin^2\theta_W(M_Z)$. In ref. [4] it was shown, in a wide range of realistic free fermionic models, that heavy string threshold corrections, non-standard hypercharge normalizations, light SUSY thresholds or intermediate gauge structure, do not resolve the disagreement with $\alpha_{\rm strong}(M_Z)$ and $\sin^2\theta_W(M_Z)$. The problem may be resolved in the superstring derived standard–like models due to the existence of color triplets and electroweak doublets from exotic sectors that arise from the additional vectors α, β and γ. For example, the model of Ref. [12] is obtained from the model of ref. [1] by a change of GSO phase that preserves the observable massless spectrum and interactions. This model contains in its spectrum two pairs of $(\overline{3},1)_{1/3}$ color triplets with beta-function coefficients $(b_3,b_2,b_1)=(1/2,0,1/5)$, one pair of $(\overline{3},1)_{1/6}$ triplets with $b_i=(1/2,0,1/20)$, and three pairs of $(1,2)_0$ doublets with $b_i=(0,1/2,0)$. This particular combination of representations and hypercharge assignments opens up a sizable window in which the low-energy data and string unification can be reconciled. For example, it is found that if these triplets all have equal masses in the approximate range $2\times10^{11}\le M_3\le 7\times10^{13}$ GeV with the doublet masses in the corresponding range $9\times10^{13}\le M_2\le 7\times10^{14}$ GeV, then the discrepancy is removed.

This extra matter at intermediate energy scales may also affect the top quark mass prediction, Eq. (1). To study this effect, I take the two–loop RGEs for the gauge and Yukawa couplings, including the contribution of the extra matter. To account for the dependence of $M_{\rm string}$ on $g_{\rm string}$, $M_{\rm string}$ is varied in the range $(3-7)\times10^{17}$ GeV. The Yukawa couplings at $M_{\rm string}$ are given by Eqs. (7,9), and $g_{\rm string}$ is varied in the range $0.03-0.07$. The two–loop RGEs are then evolved to the extra doublets and triplets thresholds. The extra doublet and triplet thresholds are varied in the ranges $1\times10^{13}\le M_2\le 1\times10^{16}$ GeV and $9\times10^{9}\le M_3\le 1\times10^{12}$ GeV, respectively. The contribution of each threshold to the β–function coefficients is removed in a step approximation. The two–loop RGEs are then evolved to the approximate top quark mass scale, $m_t\approx 175$

GeV. At this scale the top quark Yukawa coupling and $\alpha_{\rm strong}(m_t)$ are extracted, and the contribution of the top quark to the RGEs is removed. The two–loop RGEs are then evolved to the Z mass scale and agreement with the experimental values of $\alpha_{\rm strong}(M_Z)=0.12\pm0.01$, $\sin^2\theta_W(M_Z)=0.232\pm0.001$ and $\alpha_{\rm em}^{-1}(M_Z)=127.9\pm0.1$ is imposed. The bottom quark and tau lepton masses, $m_b(m_b) = 4.3\pm0.2$ and $m_\tau(m_\tau)=1777.1^{+0.4}_{-0.5}$ MeV [14] are evolved from their physical mass scale to the Z–mass scale by using the three–loop QCD and two–loop QED RGEs [13]. The bottom mass is then used to extract the running top quark mass, using Eq. (10). The physical top quark mass is given by, $m_t(physical)=m_t(m_t)(1+\frac{4}{3\pi}\alpha_{\rm strong}(m_t))$ where $m_t(m_t)$ is given by Eq. (10). It is found that the effect of the intermediate matter thresholds is to push $m_t(m_t)$ to the mass range $m_t(m_t)\approx 185-190$ GeV. The physical top quark mass is in the interval

$$m_t(physical)\approx 192-200{\rm GeV}, \tag{11}$$

which is in agreement with the CDF and D0 results.

From Eq. (9) we observe that in this model $\lambda_b=\lambda_\tau$ at the string unification scale. Consequently, an additional prediction for the mass ratio

$$\lambda_b(M_Z)/\lambda_\tau(M_Z)=m_b(M_Z)/m_\tau(M_Z) \tag{12}$$

is obtained. λ_b and λ_τ are extrapolated from the string unification scale to the Z–mass scale using the two–loop RGEs with the intermediate matter thresholds, as described above. The gauge couplings of $SU(3)_{\rm color}\times U(1)_{em}$ are then extrapolated to the bottom quark mass scale. The bottom quark and tau lepton masses are then extrapolated to the Z mass scale. It is then found that the predicted ratio of

$$\lambda_b(M_Z)/\lambda_\tau(M_Z)\approx 1.86-1.98 \tag{13}$$

is in good agreement with the extrapolated value of $m_b(M_Z)/m_\tau(M_Z)\approx 1.57-1.93$.

In this paper I have shown that LEP precision data for $\alpha_{\rm strong}$ and $\sin^2\theta_W$ as well as the CDF/D0 top quark observation and the b/τ mass relation can all simultaneously be consistent with the superstring derived standard–like models. This is achieved if

the additional matter states that are obtained in the string derived models exist at the appropriate thresholds. It will be of further interest to examine whether these extra matter states have additional testable predictions and whether the remaining fermion mass spectrum can be derived from the superstring standard–like models. Such work is in progress and will be reported in future publications.

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